The relative uncertainties in the apparent densities and permeability parameters for an individual sample were estimated from uncertainties in the contributing experimental parameters using conventional methodology.[1,2] For PICA specimens, the uncertainties in the apparent densities are about 1% for $\tilde{\rho}_{lca,v}$ and $\tilde{\rho}_{lca,c}$; 4% for $\tilde{\rho}_t$; and 8% for $\tilde{\rho}_{r,v}$. For SIRCA specimens, these uncertainties are about 10% for $\tilde{\rho}_{lca,v}$ and $\tilde{\rho}_{lca,c}$; 14% for $\tilde{\rho}_t$; and 18% for $\tilde{\rho}_{r,v}$. A detailed uncertainty analysis for K_0 and b has been given in Refs. 1 and 2. The estimated uncertainties for rigid tile specimens are +11/ −16% for K_0 and +7/ −4% for b. For the LCA materials these uncertainties are raised to +21/ −26% for K_0 and +17/ −14% for b, to account for the uncertain sample length in the case of SIRCA and the possibility of non-Darcian flow contributions[1] for PICA.

Discussion

The continuum permeabilities for transversely oriented virgin PICA and SIRCA samples are on the order of 10^{-11} m^2 and 10^{-15} m^2, respectively. Clearly, virgin SIRCA is much less permeable to gas flow than virgin PICA. PICA permeability is comparable with that of unimpregnated fibrous tile insulations such as LI-900 and FRCI-12,[2] whereas SIRCA permeability is four orders of magnitude smaller. The large difference in permeability between PICA and SIRCA can be related to differences in substrate microstructure. FiberForm is less dense than FRCI-12 and is composed of larger-diameter fibers; i.e., ∼15-μm carbon fibers vs ∼ 2-μm silica and ∼8-μm Nextel® fibers. As a result, FiberForm has a much coarser and more open microstructure than FRCI-12. The continuum permeabilities of the transversely oriented FiberForm and FRCI-12 tile substrates are on the order of 10^{-10} and 10^{-11} m^2, respectively.[2] From the experimental data, it is clear that silicone resin impregnation of FRCI-12 restricts internal gas flow much more than phenolic resin impregnation of FiberForm. There are several possible mechanisms to explain this. First, pressure-driven internal mass flow in a porous material scales to the third power (at least) with the characteristic channel dimension. Thus, comparable resin loadings into substrates with comparable densities are expected to cause greater flow restrictions in the material with the finer microstructure. Secondly, while resin impregnation predominantly coats the fibers, some webbing between fibers is possible and has been observed in scanning electron microscope images. Such webbing would be more likely in the finer microstructure substrate (FRCI-12) and may contribute to the very low continuum permeability of virgin SIRCA.

For PICA specimens, the continuum permeability is larger and the slip parameter is smaller (on average) along in-plane directions than along transverse directions. This result is consistent with the anisotropic microstructure of the FiberForm substrate, which causes the flow path to be less tortuous and the mean distance between gas-surface collisions to be longer along in-plane directions than transverse directions.[2] Though FRCI-12 substrates have microstructural anisotropies similar to FiberForm, the transverse permeabilities of the virgin SIRCA specimens are found to be comparable with or greater than the in-plane permeabilities. This finding gives further evidence that the internal flow properties of virgin SIRCA are dominated by the microstructural changes produced by resin impregnation.

As expected from the mass loss accompanying pyrolysis, charred specimens of both PICA and SIRCA offer less obstruction to the flow than virgin specimens. For PICA specimens, the relative changes in K_0 and b due to the pyrolysis process are larger for the transversely oriented specimens than the in-plane oriented specimens. This result is consonant with the view that resin is likely to agglomerate or web at fiber intersections during impregnation. Because fibers are preferentially oriented normal to the transverse direction, agglomerations at fiber intersections would offer more obstruction to transverse flow than in-plane flow, and charring would thus have a greater effect on transverse than in-plane permeability. The continuum permeability changes dramatically for SIRCA specimens upon charring, increasing by three orders of magnitude. Moreover, the underlying anisotropy of the FRCI-12 substrate is recovered; in-plane oriented specimens of charred SIRCA are about twice as permeable as transversely oriented specimens.

It is possible that the permeability of charred PICA in different heating environments, e.g., high-temperature, oxidizing atmospheres, could be greater than reported here; however, using FiberForm substrate permeability parameters as approximations for charred PICA would probably still overestimate the true char permeability. As with PICA, SIRCA char formation during a particular application will depend on the chemical environment and heating rate to which the material is exposed. For the rapid exposure of SIRCA to high heating rates in an oxidizing environment, evidence exists for the formation of a glassy char layer, which appears to seal the surface.[4] In such an application, the composite permeability of a partially charred SIRCA component would likely be closer to that of virgin SIRCA. For lower heating-rate environments, where such a glassy char layer does not form,[5] SIRCA permeability should tend toward the char values reported here.

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Effect of Nonspherical Shape on Oscillations of Levitated Droplets

Arturo A. Fuentes* and Yildiz Bayazitoglu†
Rice University, Houston, Texas 77005-1892

Nomenclature

a, b	= major and minor axis of the oblate spheroid, cm
g	= acceleration of gravity, cm/s^2
$I_{um}^{(l)}$	= integral of product of three spherical harmonics
l, m	= mode numbers
p, p_0	= perturbed and hydrostatic pressure
p_{ext}	= externally applied pressure force
R	= equivalent spherical radius, cm
R_1, R_2	= local radii of curvature of droplet surface
$r(\theta, \chi, t)$	= shape of oscillating, deformed droplet
t	= time, s
$\bar{u}$	= perturbed velocity
$X(\theta, \chi)$, $Z(\theta, \chi, t)$	= static and oscillatory deformations of droplet
$Y_{lm}(\theta, \chi)$	= spherical harmonic for mode l, m
$\alpha_1, \alpha_2, \alpha_3$	= dimensionless variables
β	= exponential coefficient of time dependence of Z
γ	= surface tension, dyne/cm

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*Graduate Student, Department of Mechanical Engineering and Materials Science, George R. Brown School of Engineering.

†Professor, Department of Mechanical Engineering and Materials Science, George R. Brown School of Engineering. Member AIAA.

Δp = change in pressure across the droplet surface
ζ_{lm}, χ_{lm} = coefficients of oscillatory and static deformation for mode l, m
θ, χ = azimuthal and polar angles in spherical coordinates
ρ = density, g/cm^3
σ = surface of droplet, surface profile
ω_{lm} = natural frequency for mode l, m, rad/s

I. Introduction

SURFACE tension determines the natural frequency of oscillation of an inviscid liquid droplet as evidenced by the Rayleigh–Lamb equation[1,2]:

$$\omega_l^{*2} = \gamma l(l-1)(l+2)/\rho R^3 \tag{1}$$

The fundamental ($l = 2$) mode represents oblate-prolate oscillations, i.e., an alternating flattening and extension of the poles of the droplet. Some other assumptions employed in Eq. (1) are medium (in which the droplet levitates) of negligible density and viscosity, amplitude of oscillations small relative to the size of the droplet, restoring force arising solely from surface tension, and no rotation of the droplet (to prevent centripetal and Coriolis forces). Whereas some investigators[3–6] have addressed the effect of viscosity on the oscillatory behavior of spherical liquid droplets, others[7–9] have focused on inviscid droplets whose equilibrium shapes are aspherical because of the external force required to counteract gravity. Few analyses have addressed the effect of the static deformation on droplet oscillations. Cummings and Blackburn[8] analyzed the behavior of the fundamental ($l = 2$) mode of an aspherical droplet deformed by an electromagnetic levitating force and demonstrated a splitting of the frequency spectrum caused by the deformation. Suryanarayana and Bayazitoglu[3,9] considered in detail the dynamics of a viscous droplet in a viscous medium. Their work presented criteria for neglecting the effect of the outer medium and the viscosity of the droplet itself. They analyzed the effect of static deformation from an arbitrary external force and applied their findings to acoustic and electromagnetic forces, calculating the frequency splitting for modes $l = 2$, 3, and 4. Bayazitoglu and Mitchell[10] and Mitchell et al.[11] presented experimental data and observations on the frequency splitting and surface tension and viscosity of acoustically levitated samples. The object of this work is to develop a general theory for the dynamics of aspherical droplets useful in accounting for interpreting frequency spectra more accurately. The effect of an arbitrary static shape deformation on the time-dependent oscillations of a liquid is considered without regard to its specific cause. The analysis is based on an inviscid, incompressible droplet oscillating freely in a medium of negligible density and viscosity.

II. Analysis

Consider an inviscid, incompressible droplet oscillating freely in a medium of negligible density and viscosity. The presumption is made that the ratio $gR^2\Delta\rho/\gamma$ is sufficiently small so that the gravity effect can be neglected. The equation governing the behavior of the droplet is

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho}\nabla p \tag{2}$$

where the equation is written for the perturbed quantities. When this equation, along with the continuity condition, is solved, one finds that

$$\nabla^2 p = 0 \tag{3}$$

When working in oblate spheroidal coordinates, finding a solution for Eq. (3) becomes complicated. In this coordinate system the Laplacian of the pressure is given by

$$\nabla^2 p = \frac{1}{b^2 + e^2\cos^2\theta}\left\{\frac{\partial}{\partial b}\left[(b^2+e^2)\frac{\partial p}{\partial b}\right] + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial p}{\partial\theta}\right] + \left(\frac{1}{\sin^2\theta} - \frac{e^2}{b^2+e^2}\right)\frac{\partial^2 p}{\partial\chi^2}\right\} \tag{4}$$

where

$$e = (a^2 - b^2)^{0.5}$$

Noticing that, for a small asphericity ($b \gg e$), the solution for the pressure is the same as in spherical coordinates, we use the solution in spherical coordinates as an approximate solution. The approximate solution is given by

$$p = p_0 + \sum_{l=1}^{\infty}\sum_{m=-l}^{l}\frac{\beta\rho}{l}a_1 r^l e^{-\beta t}Y_{lm}(\theta,\chi) \tag{5}$$

where r is the radial coordinate and Y is a spherical harmonic. In addition, the dependence on time is assumed to be of the form $e^{-\beta t}$. The real part of β is the damping or amplification factor, with the damping occurring for the positive real part. The imaginary part is the frequency of oscillations. When β is purely imaginary (such as in the inviscid case), undamped oscillations occur. The radial velocity w is obtained as

$$w = \frac{1}{r}\sum_{l=1}^{\infty}\sum_{m=-l}^{l}a_1 r^l e^{-\beta t}Y_{lm}(\theta,\chi) \tag{6}$$

In applying the boundary conditions the effect of static shape deformation as well as time-dependent oscillations must be included. Therefore the assumption is made that the perturbed radius of the sphere (at the surface) may be expressed as a function of the equivalent spherical radius R:

$$r = R + X(\theta,\chi) + Z(\theta,\chi,t) \tag{7}$$

where the static deformation $X(\theta,\chi)$ and the time-dependent distortions $Z(\theta,\chi,t)$ can be expanded in terms of independent spherical harmonics as

$$X(\theta,\chi) = \sum_{u=1}^{\infty}\sum_{v=-u}^{u}\mathcal{X}_{uv}Y_{uv}(\theta,\chi)$$
$$Z(\theta,\chi,t) = \sum_{p=1}^{\infty}\sum_{q=-p}^{p}\varsigma_{pq}e^{-\beta t}Y_{pq}(\theta,\chi) \tag{8}$$

Thus the surface of the droplet at any time is given by the equation

$$\sigma = r - [R + X(\theta,\chi) + Z(\theta,\chi,t)] = 0 \tag{9}$$

In this work the subscripts p, q, u, and v are dropped unless there is a possibility of ambiguity, and in writing the spherical harmonic functions their dependence on the angles is understood. The boundary conditions applied to the surface $\sigma = 0$ in the equations of motion are the kinematic condition and the normal force balance. The kinematic condition simply states that $w = \partial Z/\partial t$ at the surface or

$$\frac{1}{(R+X+Z)}\sum_{l=1}^{\infty}\sum_{m=-l}^{l}a_1(R+X+Z)^l e^{-\beta t}Y_{lm} = \sum_{l=1}^{\infty}\sum_{m=-l}^{l}-\varsigma_{lm}\beta e^{-\beta t}Y_{lm} \tag{10}$$

because it refers to the surface. Expanding by using the binomial theorem, neglecting terms of the order Z^2, X^3, and higher, and employing the orthogonality of spherical harmonics,

$$\iint Y_{lm}Y^*_{l'm'}\sin\theta\,d\theta\,d\chi = \delta_{ll'}\delta_{mm'} \tag{11}$$

the kinematic condition becomes

$$\frac{a_1}{R^2}R^l\left[R + (l-1)\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I_{um}^{(l)} + \frac{(l-1)(l-2)}{2R}\times\left(\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I_{um}^{(l)}\right)^2\right] = -\zeta_m\beta \tag{12}$$

where

$$I_{um}^{(l)} = \iint Y_{lm} Y_{l'm'}^{*} Y_{uv}\, ds$$

The normal force balance condition at the surface is written as

$$\Delta p = \gamma(1/R_1 + 1/R_2) \tag{13}$$

where Δp, the pressure excess at the surface, is given by Eq. (2) and

$$1/R_1 + 1/R_2 = \nabla \cdot \hat{n} \tag{14}$$

where $\hat{n}$ is the unit outward normal to the surface

$$\hat{n} = \nabla\sigma / |\nabla\sigma| \tag{15}$$

and hence

$$\nabla \cdot \hat{n} = \nabla^2\sigma/|\nabla\sigma| + \nabla\sigma \cdot \nabla(1/|\nabla\sigma|) \tag{16}$$

Using Eq. (16), $1/R_1 + 1/R_2$ can be expressed in terms of X and Z. Substituting for σ in Eq. (16) and replacing $b^2 + e^2\cos^2\theta$ by $(R + X + Z)^2$, we have

$$\nabla \cdot \hat{n} = \frac{\alpha_1}{\alpha_2(R + X + Z)} + \frac{\hat{L}^2(X + Z)}{\alpha_2(R + X + Z)^2} + \alpha_3 \tag{17}$$

where the operator

$$\hat{L}^2 = -\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) - \left(\frac{1}{\sin^2\theta} - \frac{e^2}{b^2 + e^2}\right)\frac{\partial}{\partial\chi^2}$$

$$\alpha_1 = \left[2b - \frac{3e^4\cos^4\theta}{4b^3} + \frac{e^4\cos^2\theta}{b^3} - \frac{3e^6\cos^4\theta}{2b^5} - \frac{2e^2\cos^2\theta}{b} + \frac{e^2\sin^2\theta}{b} + \frac{e^4\cos^4\theta}{b^3} - \frac{3e^4\cos^2\theta\sin^2\theta}{2b^3}\right] \Big/ (b^2 + e^2\cos^2\theta)^{0.5}$$

$$\alpha_2 = \frac{(b^2 + e^2)^{0.5}}{(b^2 + e^2\cos^2\theta)^{0.5}}\left(1 - \frac{e^2\cos^2\theta}{2b^2} + \frac{3e^4\cos^4\theta}{8b^4}\right)$$

For the surface under consideration, Eq. (16), after neglecting higher-order terms, becomes

$$\nabla \cdot \hat{n} = \frac{\alpha_1}{\alpha_2 R}\left[1 - \frac{X}{R} - \frac{Z}{R} + \frac{X^2}{R^2} + \frac{2XZ}{R^2} - \frac{3X^2Z}{R^3}\right] + \frac{1}{\alpha_2 R^2}\left[\hat{L}^2X + \hat{L}^2Z - \frac{2X\hat{L}^2X}{R} - \frac{2Z\hat{L}^2X}{R} - \frac{2X\hat{L}^2Z}{R} + \frac{6XZ\hat{L}^2X}{R^2} + \frac{3X^2\hat{L}^2X}{R^2}\right] + \alpha_3 \tag{18}$$

In arriving at Eqs. (12) and (18), terms of order X^3, Z^2, and higher have been neglected, which is justified for a small-amplitude oscillation and a moderate static deformation. Notice, however, that terms of order XZ have been retained. The XZ terms are the first-order effect of the static shape deformation on the time-dependent oscillation terms. If XZ terms were to be neglected, the problem would separate into a static part and a time-dependent part, each independent of the other. In such a case, the static deformation and the oscillations can both be calculated, but the solution would fail to reveal the influence of this static deformation on the oscillations. Because XZ represents the interrelation of the static and dynamic parts of the problem, the XZ terms must be retained if this effect is of concern, as it is in this work.

Now substituting Eqs. (5) and (18) into Eq. (13), simplifying, and employing the orthogonality of spherical harmonics, the time-dependent part becomes

$$\frac{\beta\rho a_1}{l\alpha_4}R^{l-1}\left\{R + l\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I_{um}^{(l)} + \frac{l(l-1)}{2R}\times\left[\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I_{um}^{(l)}\right]^2\right\} = \frac{\gamma\zeta_m}{\alpha_2R^2}\Bigg(-\alpha_1 + \frac{2\alpha_1}{R}\times\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I_{um}^{(l)} - \frac{3\alpha_1}{R^2}\left[\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I_{um}^{(l)}\right]^2 + l(l-2)\alpha_5 + l\alpha_6 - \frac{2}{R}\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I_{um}^{(l)}\times\{[l(l-2) + u(u-2)]\alpha_5 + (l+u)\alpha_6\} + \frac{3}{R^2}\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I_{um}^{(l)}\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I_{um}^{(l)}\times\{[l(l-2) + 2u(u-2)]\alpha_5 + [l + 2u]\alpha_6\}\Bigg) \tag{19}$$

where

$$\alpha_4 = \frac{\partial r}{\partial b}, \qquad \alpha_5 = \frac{b^4 + e^2b^2}{(b^2 + e^2\cos^2\theta)^2}$$

$$\alpha_6 = \frac{3b^2 + e^2}{b^2 + e^2\cos^2\theta}$$

In Eqs. (12) and (19) we have integrals of the products of three spherical harmonics of the form

$$\iint Y_{lm}Y_{l'm'}^{*}Y_{uv}\, ds \tag{20}$$

in which integration is over all solid angles. In this work these integrals are evaluated using the Graunt formula in terms of the Wigner 3-j symbols.

III. Results and Discussion

A. Frequencies of Oscillation

For an inviscid droplet β is purely imaginary, and Eqs. (12) and (19) must be solved simultaneously to obtain ω_{lm}. Eliminating a_1 between the two equations, expanding using the binomial theorem, neglecting terms of order X^3 and higher, and noting that $\alpha_4 = 1$, the frequencies of oscillation are obtained as

$$(\omega_{lm})^2 = \frac{\gamma l}{\rho R^3\alpha_2}\Bigg(-\alpha_1 + l(l-2)\alpha_5 + l\alpha_6 + \sum_{u=1}^{\infty}\sum_{v=-u}^{u}\frac{X_{uv}I_{um}^{(l)}}{R}\{3\alpha_1 - [3l(l-2) + 2u(u-2)]\alpha_5 + (3l+2u)\alpha_6\} + \sum_{u=1}^{\infty}\sum_{v=-u}^{u}\frac{X_{uv}I_{um}^{(l)}}{R^2}\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I_{um}^{(l)}\{-6\alpha_1 + [6l(l-2) + 8u(u-2)]\alpha_5 + (6l + 8u)\alpha_6\}\Bigg) \tag{21}$$

Before attempting to calculate the frequencies for any mode, a limiting condition is applied to Eq. (21). When there is no static deformation (e and X are zero), the solution reduces to Eq. (1), as expected.

B. Inclusion of Surface Force

Assume an arbitrary force that acts at the surface of the droplet and causes a distortion of the droplet static shape. We assume that the interaction between the drop oscillation and the external field is negligible. In general, the force can be expanded in terms of spherical harmonics as

$$P_{\text{ext}} = \sum C^n_{l,m}\left(\frac{R+X+Z}{R}\right)^n Y_{lm}(\theta, \chi) \tag{22}$$

This is a Taylor-series expansion of an arbitrary function about the origin in spherical coordinates, and P_{ext} is assumed to be sufficiently continuous. The surface condition

$$\delta p = \gamma(1/R_1 + 1/R_2) + P_{\text{ext}} \tag{23}$$

can now be applied. The time-dependent terms in the complete surface condition give after simplification

$$\begin{aligned}
(\omega_{lm})^2 = \frac{\gamma l \alpha_4}{\rho R^3 \alpha_2}\Bigg(& -\alpha_1 + l(l-2)\alpha_5 + l\alpha_6 \\
& + 2\sum_{u=1}^{\infty}\sum_{v=-u}^{u}\frac{X_{uv}I^{(l)}_{um}}{R}\{\alpha_1 - [l(l-2) + u(u-2)]\alpha_5 \\
& + (l+u)\alpha_6\} + 3\sum_{u=1}^{\infty}\sum_{v=-u}^{u}\frac{X_{uv}I^{(l)}_{um}}{R^2}\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I^{(l)}_{um}\{-\alpha_1 \\
& + [l(l-2) + 2u(u-2)]\alpha_5 + [l+2u]\alpha_6\} + \sum\frac{n}{R}C^n_{uv}I^{(l)}_{um}\Bigg) \\
\times \Bigg(1 - \Bigg\{ & \sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I^{(l)}_{um} + [(l-1)/R] \\
& \times\left[\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I^{(l)}_{um}\right]^2\Bigg\} \Bigg/ \Bigg\{R + l\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I^{(l)}_{um} \\
& + [l(l-1)/2R]\left[\sum_{u=1}^{\infty}\sum_{v=-u}^{u}X_{uv}I^{(l)}_{um}\right]^2\Bigg\}\Bigg)
\end{aligned} \tag{24}$$

IV. Conclusions

In this work the oscillations of an oblate spheroid shape inviscid droplet subjected to external forces have been considered. The analysis developed can be extended to consider different static deformation shapes. We have explained the frequency splitting frequently observed in experiments and are able to interpret these split spectra more accurately in thermophysical property measurements. Comparisons between experiment data and analytical predictions concerning the frequency splitting of deformed levitating droplets are instructive for evaluating the state of knowledge of the forces acting on a levitated droplet.

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Interaction of Radiation with Natural Convection

C. K. Krishnaprakas* and K. Badari Narayana†
ISRO Satellite Centre, Bangalore 560 017, India
and
Pradip Dutta‡
Indian Institute of Science, Bangalore 560 012, India

Nomenclature

f	= dimensionless stream function, $(Gr/4)^{-1/4}\psi(x, y)/(4\nu)$
Gr	= Grashof number, $g\lambda\|T_w - T_\infty\|x^3/\nu^2$
g	= acceleration caused by gravity, m/s^2, dimensionless intensity function
$I_b(T)$	= blackbody radiation intensity, W/m^2-sr
$I^+(\upsilon, \mu, \xi)$	= radiation intensity for positive values of μ, W/m^2-sr
$I^-(\upsilon, -\mu, \xi)$	= radiation intensity for negative values of μ, W/m^2-sr
k	= thermal conductivity, W/m-K
N_c	= conduction-radiation number $= k\beta/4\sigma T_w^3$
Pr	= Prandtl number, ν/α
$p(\mu, \mu')$	= slab-scattering phase function
$p(\mu_p)$	= single-scattering phase function
Q^r	= dimensionless radiative heat flux, $q^r/4\sigma T_w^4$
q^r	= radiative heat flux, W/m^2
q^t_w	= total heat flux from wall, W/m^2
T	= temperature, K
T_w	= hot-plate temperature, K
T_∞	= ambient temperature, K
u	= x-direction velocity component, m/s $= \partial\psi/\partial y$
v	= y-direction velocity component, m/s $= -\partial\psi/\partial x$
x, y	= coordinates respectively in the parallel and perpendicular directions of the plate, m
α	= thermal diffusivity, m^2/s

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*Engineer; currently Research Scholar, Indian Institute of Science, Bangalore 560 012, India.

†Head, Thermal Design Section, Thermal Systems Group.

‡Assistant Professor, Mechanical Engineering Department.